FACTORIZATION IN COLOR-SUPPRESSED $\bar{B} \to D^{(*)}\pi$ DECAYS FROM THE SOFT-COLLINEAR EFFECTIVE THEORY

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The soft-collinear effective theory has been recently applied to prove novel factorization theorems for many B decay processes. We describe here in some detail the factorization relation for color-supressed nonleptonic $B \to D^{(*)0}\pi^0$ decays and update the phenomenological analysis of these decays.

1 Factorization and SCET

The application of factorization to exclusive processes has been around now for 25 years (see [1] for a review of the early literature). These factorization theorems have been traditionally proved using diagrammatic methods in perturbation theory. Apart from being rather involved technically, this approach has met with very limited success beyond leading order in the $1/Q$ expansion, where power suppressed terms generally give rise to divergent convolution integrals.

Recently an alternative approach to factorization in exclusive processes has been proposed, based on the soft-collinear effective theory (SCET) [2]. This greatly simplifies the proof of factorization theorems and allows a systematic treatment of power corrections.

The SCET is constructed as a systematic expansion in $\lambda = \Lambda/Q \ll 1$, where Q is a large scale specific to the problem, such as $Q^2 = -q^2$ in the electromagnetic π form factor at a space-like momentum transfer, or $Q \sim m_b$ in the heavy quark decay, and $\Lambda \sim 500$ MeV is the QCD scale. This is achieved by identifying the relevant energy scales and the corresponding degrees of freedom. The Lagrangian and operators of SCET (such as currents) are organized in a series in λ as $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \cdots$, etc.

The exclusive processes considered here receive contributions from 3 well-separated scales $Q^2 \gg \Lambda Q \gg \Lambda^2$. This requires the introduction of a sequence of effective theories

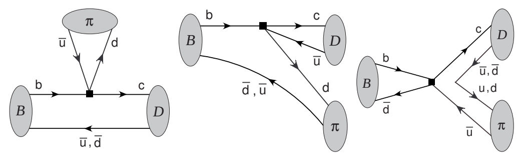

Figure 1. $\bar{B} \to D\pi$ topologies: T, C, and E. Only C and E contribute to color suppressed decays.

QCD $\to$ SCET$_{\rm I}$ $\to$ SCET$_{\rm II}$, which contain degrees of freedom of successively lower virtuality. The intermediate theory SCET$_{\rm I}$ contains hard-collinear quarks and gluons with virtuality $p_{\rm hc}^2 \sim \Lambda Q$ and ultrasoft partons with virtuality Λ^2. The final theory SCET$_{\rm II}$ includes only soft and collinear modes with virtuality $p^2 \sim \Lambda^2$.

Using the SCET many new factorization theorems were derived (see e.g. [3] for other recent applications). I will describe in this talk a factorization theorem for exclusive color-suppressed $B^0 \to D^0\pi^0$ decays [5].

2 Factorization in color-suppressed $B \to D\pi$ decays

The $\bar{B} \to D\pi$ decays are mediated by the weak Hamiltonian

$$H_w = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \big[C_1 (\bar{c}b)(\bar{d}u) + C_2 (\bar{c}_i b_j)(\bar{d}_j u_i) \big] ,$$

which can contribute through the flavor contractions shown in Fig. 1. Denoting the amplitudes as $A_{+-} = A(\bar{B}^0 \to D^+\pi^-)$, $A_{0-} = A(B^- \to D^0\pi^-)$, and $A_{00} = A(\bar{B}^0 \to D^0\pi^0)$, one has the decomposition into graphical and

isospin amplitudes [4]

$$\begin{aligned} A_{+-} &= T + E = \frac{1}{\sqrt{3}} A_{3/2} + \sqrt{\frac{2}{3}} A_{1/2}\,, \\ A_{0-} &= T + C = \sqrt{3}\, A_{3/2}\,, \qquad (1) \\ A_{00} &= \frac{C - E}{\sqrt{2}} = \sqrt{\frac{2}{3}} A_{3/2} - \frac{1}{\sqrt{3}} A_{1/2}\,. \end{aligned}$$

The color-allowed amplitudes A_{+-} and A_{0-} are described by a factorization theorem [6,7,8], proven with SCET in [9]

$$A^{(*)} = N^{(*)}\, \xi(w_0) \int_0^1 dx\, T^{(*)}(x,\mu)\, \phi_\pi(x) + \ldots,$$

where $\xi(w_0)$ is the Isgur-Wise function at maximum recoil, $\phi_\pi(x)$ is the light-cone distribution function for the pion, $T = 1 + O(\alpha_s)$ is the hard scattering kernel, and $N^{(*)} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* E_\pi f_\pi \sqrt{m_{D^{(*)}} m_B}(1 + m_B/m_{D^{(*)}})$. The ellipses in Eq. (2) denote terms suppressed by Λ/Q where $Q = \{m_b, m_c, E_\pi\}$. The predictions from Eq. (2) are in good experimental agreement with data on color allowed decays.

Large N_c QCD gives an alternative justification for factorization in color allowed decays. Possible tests for the underlying factorization mechanism use decays into multibody states $B \to D^{(*)} n\pi$ [13], isospin analyses of such decays [15,16] and decays into hadrons with exotic quantum numbers [14].

The color-suppressed amplitude A_{00} gets contributions from C and E, but not T. Large N_c predicts $C/T \sim E/T \sim 1/N_c$ (counting $C_1 \sim 1$ and $C_2 \sim 1/N_c$). Writing the isospin relation among amplitudes as

$$R_I = 1 - \frac{3}{\sqrt{2}} \frac{A_{00}}{A_{0-}}\,, \qquad (2)$$

where $R_I = A_{1/2}/(\sqrt{2} A_{3/2}) \equiv |R_I| e^{i\delta_I}$, the suppression of C, E is measured by the deviation of R_I from 1 in the complex plane [10]. We show in Table I the present experimental situation for $\bar{B} \to D^{(*)}\pi$ decays, together with the corresponding results for R_I.

SCET gives a quantitative description of the color suppressed amplitude, expressed as

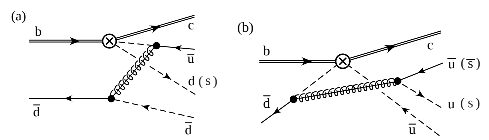

Figure 2. Diagrams in $\mathrm{SCET_I}$ for tree level matching. $\otimes$ denotes the operator $Q_j^{(0,8)}$ and the dots are insertions of $\mathcal{L}_{\xi q}^{(1)}$. The solid lines and double solid lines carry momenta $p^\mu \sim \Lambda$ and form the B and D. The dashed lines are energetic collinear quarks that form the light meson M.

a factorization theorem for A_{00}. The main observation is that in $\mathrm{SCET_I}$ these decays are mediated by only one type of operators [5] $\mathcal{H}_W \to T(Q_j^{(0,8)}(0), i\mathcal{L}_{\xi q}^{(1)}(x), i\mathcal{L}_{\xi q}^{(1)}(y))$. By power counting it follows that these decays are power suppressed by Λ/Q, with $Q = \{m_c, m_b, E_\pi\}$. This T-product contributes at tree level as shown in Fig. 2.

When matched onto $\mathrm{SCET_{II}}$ the time-ordered product gives a product of soft $O_s^{(0,8)}$ and collinear O_c operators, where the soft operators are

$$O_s^{(0)} = (\bar{h}_{v'}^{(c)} S) \not{n} P_L\, (S^\dagger h_v^{(b)})\, (\bar{d}\, S)_{k_1^+} \not{n} P_L\, (S^\dagger u)_{k_2^+}\,,$$

and $O_s^{(8)}$ is identical with color structure $T^a \otimes T^a$. In addition there are operators encoding long-distance contributions in $\mathrm{SCET_{II}}$ coming from the region of momentum space in Fig. 2 with a hard gluon ($p_g^2 \sim Q\Lambda$), but an on-shell quark propagator ($p_q^2 \sim \Lambda^2$).

Heavy quark symmetry relates the $\bar{B} \to D$ and $\bar{B} \to D^*$ matrix elements of $O_s^{(0,8)}$ as

$$\langle D^{(*)} | O_s^{(0,8)} | B \rangle = \sqrt{m_B m_D^{(*)}}\, S_L^{(0,8)}(k_1^+, k_2^+), \qquad (3)$$

The complete factorization theorem for color suppressed decays can now be written as [5]

$$\begin{aligned} A_{00}^{D^{(*)}M} = N_0^M \int & dx\, dz\, dk_i^+\, T_{L\mp R}^{(i)}(z) \qquad (4) \\ & \times J^{(i)}(z, x, k_i^+) S^{(i)}(k_1^+, k_2^+)\, \phi_M(x) + A_{long}^{D^{(*)}M}\,. \end{aligned}$$

with $T_{L\mp R}^{(i)}$ hard scattering kernels and $N_0^M = \frac{G_F}{2} V_{cb} V_{ud}^* f_M \sqrt{m_B m_{D^{(*)}}}$. The nonperturbative dynamics is encoded in ϕ_M, the light-cone distribution function of the light

Decay	Br(10^{-3})	$(\|R_I\|, \delta_I)$	Decay	Br(10^{-3})	$(\|R_I\|, \delta_I)$
$\bar{B}^0 \to D^+\pi^-$	2.76 ± 0.25	0.70 ± 0.07	$\bar{B}^0 \to D^{*+}\pi^-$	2.76 ± 0.21	0.76 ± 0.07
$B^- \to D^0\pi^-$	4.98 ± 0.29	$28.1^\circ \pm 3.3^\circ$	$B^- \to D^{*0}\pi^-$	4.6 ± 0.4	$31.9^\circ \pm 4.5^\circ$
$\bar{B}^0 \to D^0\pi^0$	0.260 ± 0.022		$\bar{B}^0 \to D^{*0}\pi^0$	0.27 ± 0.05	

Table 1. Experimental data on $\bar{B} \to D^{(*)}\pi$ and the corresponding results for the ratio of isospin amplitudes $R_I = A_{1/2}/(\sqrt{2}A_{3/2}) \equiv |R_I|e^{i\delta_I}$. The data is taken from [12], except for $D^0\pi^0$ which is the average of [11]. We use $\tau(B^+)/\tau(B^0) = 1.086 \pm 0.017$ [12].

meson M, and two soft functions for the $B \to D^{(*)}$ transition $S^{(0,8)}(k_i^+)$ with k_i^+ the light spectator momenta. The jet function $J^{(i)}$ appears in the matching $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$ and contains the effects from scales $p_{\rm hc}^2 \sim E_\pi \Lambda$. There is a further simplification for M an isovector meson such as π, ρ, for which the amplitude A_{long} vanishes from G invariance.

We discuss next several important implications of this factorization theorem. The soft functions $S_L^{(0,8)}$ are complex, encoding final state interactions arising from the soft Wilson line along n in the definition of the soft operators. This is a source of nonperturbative final state interaction effects, similar to those producing single spin asymmetries [21] in semi-inclusive DIS.

Second, since heavy quark symmetry relates the soft functions in $B \to D^{(*)}$, one predicts a relation between these decays

$$\delta_I(D^*\pi) = \delta_I(D\pi) + O(\alpha_s(Q), \frac{\Lambda}{Q})\,, \quad (5)$$
$$A_{00}^{D^*\pi} = A_{00}^{D\pi} + O(\alpha_s(Q), \frac{\Lambda}{Q})\,.$$

We emphasize that heavy quark symmetry alone would not have been sufficient to make this prediction, which requires soft-collinear factorization as a crucial ingredient. The current experimental data (see Table I) is in good agreement with Eq. (5).

Additional predictions are possible by expanding the jet functions in perturbation theory and working at leading order in $\alpha_s(\mu_C)$ with $\mu_C^2 = \Lambda Q$. The convolution integral over x can now be performed exactly which gives

$$A_{00}^{D^{(*)}M} = \frac{G_F}{2} V_{cb} V_{ud}^* f_\pi \sqrt{m_B m_D} \quad (6)$$
$$\times (C_1 + \frac{C_2}{N_c}) \frac{4\pi C_F \alpha_s(\mu_C)}{N_c} s_{\rm eff} \langle x^{-1}\rangle_M\,,$$

with $\langle x^{-1}\rangle_M = \int dx \frac{\phi_M(x)}{x}$, and $s_{\rm eff} = -s^{(0)} + C_2/[N_c C_F(C_1 + C_2/N_c)]\, s^{(8)}$ with $s^{(0,8)}(\mu) = -\int dk_1^+ dk_2^+ \frac{1}{k_1^+ k_2^+}\, S^{(0,8)}(k_i^+, \mu)$. Corrections to Eq. (6) are $O(\alpha_s(\mu_C), \Lambda/Q) \simeq 30\%$. The strong phase $\mathrm{Arg}(A_{00}/A_{-0})$ comes from $s_{\rm eff}$ and is thus independent on the light meson. This predicts the universality of the strong phase $\phi \equiv \mathrm{Arg}(A_{00}/A_{0-})$ for $D^{(*)}\pi$ and $D^{(*)}\rho$. The data in the Table I gives $s_{\rm eff} \equiv |s_{\rm eff}|e^{i\phi}$ with $(|s_{\rm eff}|, \phi) = (429 \pm 18\ \mathrm{MeV}, \pm(38^\circ \pm 12^\circ))$ at $\mu_c = 2.31$ MeV where $\alpha_s(\mu_C) = 0.25, C_1 = 1.15, C_2 = -0.362$. We used here $|V_{cb}| = 41.9 \times 10^{-3}$.

Finally, the relation (6) gives the leading deviation from 1 of the ratios of decays into charged pions

$$R_c^{DM} \equiv \frac{A(\bar{B}^0 \to D^+ M^-)}{A(B^- \to D^0 M^-)} \quad (7)$$
$$= 1 - \frac{16\pi\alpha_s(\mu_C) m_D}{9(m_B + m_D)E_M} \frac{\langle x^{-1}\rangle_M}{\xi(\omega_0)} s_{\rm eff}\,.$$

Predictions for color-suppressed decays using other methods have been discussed in Refs. [20].

3 Conclusions

This talk described the application of SCET to derive a new factorization relation for color suppressed decays $\bar{B} \to D^{(*)}\pi$. The result is different from the usual naive factorization *Ansatz* for color suppressed amplitudes [22] and depends on a new nonperturbative function describing the $B \to D$ transition.

Similar factorization relations were derived for other B decays into charm: nonleptonic baryon decays $\Lambda_b \to \Sigma_c \pi$ [23], decays into orbitally excited states $\bar{B}^0 \to D^{**0}\pi^0$ [24], and decays into isosinglet states with an $\eta^{(\prime)}, \omega, \phi$ [25].

Nonleptonic B decays to heavy-light states with charm have a rich phenomenology, and the factorization theorem (4) can be expected to be an useful tool to organize and explore the implications of this data for the nonperturbative structure of the heavy quark systems.

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